FREE-CONVECTION HEAT TRANSFER IN A SPHERICAL VOLUME

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Convective heat transfer in a completely filled spherical volume with a constant heat flux density on its boundary is investigated.

NOTATION

R — radius of vessel
m — mass
q — specific heat flux
c_p — specific heat at constant pressure
a — thermal diffusivity
t — temperature
z — vertical distance from lowest point of sphere
Δt — time-averaged temperature drop in boundary layer for given angle θ
α — heat transfer coefficient
β — thermal expansion coefficient
θ — coordinate angle,
λ — thermal conductivity
ν — kinematic viscosity
τ — time
F — Fourier number
N — Nusselt number
G* — modified Grashof number
P — Prandtl number

$$F=\frac{a\tau}{R^2},\quad N=\frac{qR}{\lambda\Delta t},\quad G^*=\frac{g\beta R^4 q}{\nu^2\lambda},\quad P=\frac{\nu}{a}.$$

There have been few investigations of convective heat transfer in spherical volumes. There is the work of Schmidt [1], who investigated convection in a sphere filled with ethanol, butanol, methanol, or water with a constant temperature on the boundary, and that of Ostromov [2], who investigated convection in a water-filled spherical cavity in a block containing a heater and a cooler.

In this paper the experiments were conducted in a spherical vessel of diameter 300 mm containing distilled water with an initial temperature of 20 or 50° C, or ethyl alcohol (concentration 96%) with an initial temperature of 20° C. A diagram of the apparatus is shown in Fig. 1.

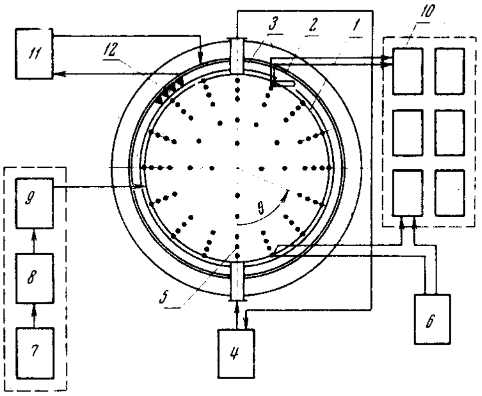

Fig. 1. Diagram of apparatus: 1) working volume with heater; 2) guard heater shell; 3) heat-insulating case; 4) TS-24 thermostat; 5) thermocouples; 6) cold-junction thermostat; 7) S-3S voltage stabilizer; 8) VSA-5 rectifier; 9) D-57 wattmeter; 10) temperature-measuring unit; 11) guard-heater power regulator; 12) thermopile.

A prescribed heat flux on the surface of the sphere was provided by an electric heater consisting of a multilayer coil of constantan wire (0.25 mm in diameter) in silk insulation impregnated with heat-resistant lacquer. The gap between the turns of the heater was about 0.1 mm. The heater was supplied with direct current from a stabilized power supply. The power was measured by a Class 1 D-57 electrostatic wattmeter. The shell of the vessel was made of thin (about 0.6 mm) material of relatively low thermal conductivity (Cr18Ni9Ti steel), which in conjunction with the design of the heater ensured a fast response of the system and a heat flux density which was constant over the whole sphere and the same for the external and internal surfaces of the shell. A preliminary estimate showed that the quantity of heat flowing through the shell in the region of the maximum temperature gradient did not exceed 1% of the quantity of heat entering this region of the shell from the heater. To prevent leakage of heat to the surroundings the investigated volume was surrounded by plastic foam insulation and furnished with an automatically controlled guard heater. The experiments could be carried out without the guard heater, since the heat loss was low (not more than 5–10%) and could be allowed for in the treatment of the experimental results.

The temperature in the liquid and on the shell was measured by copper-constantan thermocouples, whose thermoelectrodes had a diameter of 0.1 mm and were covered with teflon insulation. The emf of the thermocouples was recorded by an EPP-09 automatic multipoint potentiometer with ranges 0–400, 0–1000, and 0–3000 μW. Check measurements were made with a R-306 potentiometer and on M17/1 galvanometer.

The arrangement of the thermocouple junctions in the investigated volume is shown schematically in Fig. 1. The temperature was measured along the vertical axis of the sphere, in several horizontal sec-

Table 1

C_2H_5OH, $q = 8.4\cdot10^2$ W/m²					H_2O, $q = 3.4\cdot10^2$ W/m²				
θ°	$\Delta t'$	$\Delta t''$	$\Delta t'''$	$\alpha_1\cdot10^{-2}$	θ°	$\Delta t'$	$\Delta t''$	$\Delta t'''$	$\alpha_2\cdot10^{-2}$
0	4.2	4.7	4.7	1.78	0	1.2	1.3	1.3	2.62
22.5	4.6	4.8	4.8	1.75	22.5	1.1	1.2	1.2	2.84
45	4.1	4.7	4.7	1.78	45	1.3	1.3	1.4	2.62
67.5	3.9	4.9	4.9	1.71	67.5	0.8	1.2	1.3	2.84
112.5	3.8	5.0	5.0	1.68	112.5	0.8	1.2	1.2	2.84
135	3.7	4.5	5.0	1.68	135	1.0	1.2	1.4	2.62
157.5	3.5	3.8	5.3	1.58	157.5	1.1	1.3	1.5	2.27
180	3.9	4.6	6.0	1.40	180	0.9	1.4	1.7	2.00

Note. $\alpha^* = 1.6\cdot10^2$ W/m² · deg; $\alpha_2^* = 2.5\cdot10^2$ W/m² · deg tions, and also at intervals of 22.5° in a meridional section in the region of the thermal boundary layer and on the internal surface of the shell. Looking ahead, we note that the temperature distributions in the investigated process, with the limits of accuracy of temperature measurement (0.1–0.3°), were symmetric relative to the vertical axis and, hence, the results obtained for the meridian section were valid for the whole volume. In the boundary layer the thermocouple junctions were situated along a normal to the shell surface at a given coordinate angle θ at distances of 1, 2, and 5 mm from it and were connected differentially with the corresponding thermocouple on the shell. This enabled us to record the temperature drops across the boundary layer by means of narrow-range EPP-09 instruments with a scale of 0–400 μV. The thermoelectrodes, contained in a metal capillary, were situated in the equitemperature region at a distance of 30 mm from the junction. This significantly reduced the error in temperature measurement due to flow of heat through the electrodes to the junction.

Before the start of the experiment the temperature of the investigated volume was made constant by repeated pumping of the working liquid through it and the TS-24 thermostat. When the liquid had settled a constant power was applied to the heater and the change in temperature at different points in the volume was automatically recorded. The experiments lasted 1 to 10 hr, depending on the heat flux density, which varied in the investigations from $1.7 \cdot 10^2$ to $1.8 \cdot 10^3$ W/m^2.

The investigations showed that heat transfer on the wall-liquid boundary in a spherical volume with the indicated boundary conditions can be regarded as quasistationary, since a short time after the start of the process there is a steady time-averaged* temperature drop in the boundary layer for each point on the boundary and the heating curves of points at the same level in the liquid and on the shell are parallel straight lines. The period in which the formation of the boundary layer took place in the discussed experiments corresponded to a Fourier number $F \approx 4 \cdot 10^{-4}$.

The quasistationarity of heat transfer at the wall-liquid boundary allows us to exclude the Fourier number from the characteristic criteria in finding the relationship for the heat transfer coefficients and to take

$$N = f(G^*, P).$$

If the criterion G does not exceed 10^{10} in order of magnitude, the effect of inertial forces [3,4] in free-convection conditions can be regarded as small,

$$N = C(G^*P)^n \qquad (C,\ n = \text{const}) \tag{2}$$

An analysis of the curves of temperature distribution over the thickness of the boundary layer in different regions of the boundary indicates that the rate of heat transfer on the greater part of the wall-liquid boundary is practically the same, since we observed no significant difference in the boundary layer thickness nor in the temperature drop through the layer (Table 1). An exception is the upper region of the sphere, where the heat transfer coefficient is reduced by 10–15% at θ = 180°. In view of what has been said we can represent the heat transfer at the boundary by an integral heat transfer coefficient defined in the following way:

$$\alpha^* = \frac{1}{n} \sum_{i=1}^{n} \frac{q}{\Delta t_i(\theta)}\,. \tag{3}$$

Here $\Delta t_i(\theta)$ is the time-averaged temperature drop across the boundary layer for a given angle θ; n is the number of measurement points.

Table 1 gives the temperature drops across the boundary layer in various regions of the boundary, and the local and integral heat transfer coefficients for two heat flux densities.

Here Δt', Δt", and Δt‴ are the temperature drops (in deg) across the boundary layer at distances of 1, 2, and 5 mm from the shell, respectively; α is the local heat transfer coefficient; q is the heat flux density.

In generalized coordinates the relationship for the integral heat transfer coefficient, shown in Fig. 2, has the form

$$N = 0.83(G^*P)^{0.2} \quad \text{for} \quad 3.5 \cdot 10^9 \leqslant G^*P \leqslant 3.2 \cdot 10^{11}.$$

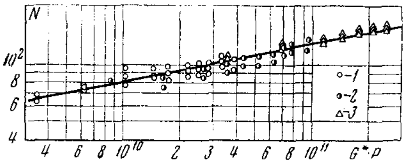

Fig. 2. 1) Water (t = 30° C); 2) water (t = = 60° C); 3) ethanol (t = 30° C).

Relationship (4) was obtained by treatment of more than 400 experimental points by the method of least squares. The mean scatter of the experimental data from the curve in Fig. 2 is within ±5%. The coefficient C and the index n are determined with the same accuracy. Owing to the absence of published formulas for the heat transfer coefficient in a spherical volume with boundary conditions of the second kind we cannot make comparisons with the results of other authors.

Owing to the nonuniformity of the temperature distribution in the volume a knowledge of the heat transfer coefficient is not sufficient for calculation of the temperature distribution on the shell bounding the volume. In the case of a spherical volume with a uniform and steady heat flux on the boundary it is sufficient to know the temperature distribution along the vertical axis, since in any horizontal cross section (apart from the boundary layer) the temperature is the same.

Figure 3 shows the temperature distribution (for water) on the vertical axis of the spherical volume and in a meridional cross section on the shell for different heat flux densities. As the figure shows, in the upper region of the sphere (z > 1.5R) there is pronounced temperature stratification of the liquid. In the lower part of the sphere (z > 1.5R) the mean temperature gradient

$$\frac{\Delta t_z}{R} = \frac{t_{\max} - t_{\min}}{R}$$

is 0.02–0.1 deg/cm, whereas in the upper region it is 0.5–2 deg/cm, i.e., 20 times higher. This pronounced thermal stratification of the liquid can be attributed to the gravitational thermal effect of convection, i.e., the temperature of the upper region of the sphere depends not only on the heat flux entering the liquid in the given region through the shell, but also on the heated mass of liquid rising along the shell from below.

The rate of heating of the points belonging to the upper surface and points lying below z = 1.5R, and also the time for attainment of the linear region of the relationship t = f(τ), is different. For the lower

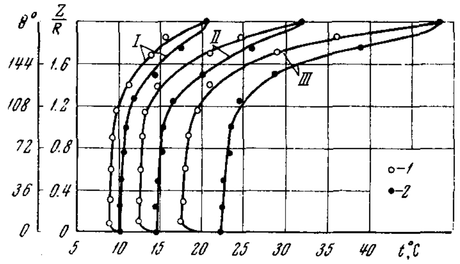

Fig. 3. 1) Liquid; 2) wall. I) q = $1.7 \cdot 10^2$ W/m^2 F = $0.8 \cdot 10^{-1}$; II) q = $5.0 \cdot 10^2$ W/m^2; F = $0.3 \cdot 10^{-1}$; III) q = $8.4 \cdot 10^2$ W/m^2; F = $0.9 \cdot 10^{-2}$.

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*A time-averaged temperature drop in this case means the temperature drop averaged over a period much shorter than the time of the experiment. We had to consider the time-averaged temperature drop in the boundary layer because of the observed temperature oscillations (with a period not exceeding 60 sec), which reached 30% of the total drop in the region 0° ≤ θ ≤ 22.5° and, in addition, with increase in the coordinate angle gradually decreased to 10–20% at θ = 67.5%; in the region 112.5° ≤ θ ≤ 180° there were hardly any oscillations. The investigation of the nature of these oscillations is beyond the scope of this work.

Table 2

№	$\frac{z}{R}$	$F_1 = 0.59 \cdot 10^{-2}$		$F_2 = 1.57 \cdot 10^{-2}$	
		$A \cdot 10^2$	k	$A \cdot 10^2$	k
1—3	0.1—0.6	1	0	3.2	0
4	0.9	1.2	0	3.5	0
5	1.15	1.5	0	3.8	0
6	1.4	3.4	—0.02	9	—0.02
7	1.7	73	—0.12	152	—0.12
8	2	146	—0.12	246	—0.12

region the time in our experiments corresponded to a Fourier number $F = 1.6 \cdot 10^{-3}$, whereas for the upper region $F = (3.5\text{–}4) \cdot 10^{-3}$. Heating of the lower region is of a quasi-stationary nature, i. e., beginning at the instant $F = 1.6 \cdot 10^{-3}$ a steady vertical temperature stratification of the liquid is established. The rate of heating of points in the region is similar to the corresponding characteristic of the mean bulk temperature.

The mean bulk temperature of the liquid was calculated by the integral method from the curve of temperature distribution along the vertical axis of the sphere and from the heat balance equation

$$t = t_0 + \frac{qS\tau}{c_p m}.$$

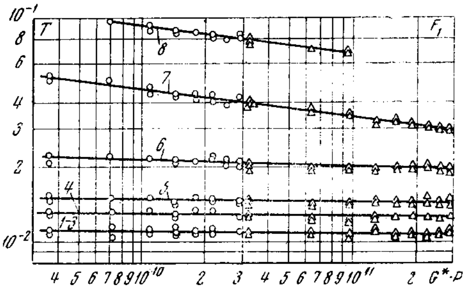

Fig. 4a. The circles are for water and the triangles for ethanol.

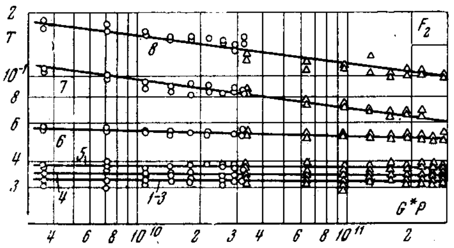

Fig. 4b.

Here t is the mean bulk temperature, t_0 is the initial temperature, S is the surface of the spherical volume, c_p is the specific heat at constant pressure, and m is the mass of the liquid.

We found that the cross section at which the liquid temperature is equal to the mean bulk temperature lies at a height $z = (1.25\text{–}1.3)R$, irrespective of the heat load on the boundary of the volume. The temperature in the greater part of the sphere ($0 \leq z \leq 1.5R$) is close to the mean bulk temperature and the deviation from t is 1–4° for a total vertical drop of 10–35°. We attempted to represent the temperature distribution along the vertical axis of the sphere in dimensionless form

$$T = A(G^*P)^k, \quad T = (t - t_0)\frac{\lambda}{qR} \quad \text{for} \quad \frac{z}{R} = \text{const} \qquad (5)$$

for two Fourier numbers: $F_1 = 0.59 \cdot 10^{-2}$ and $F_2 = 1.57 \cdot 10^{-2}$.

The values of A and k for different z/R are given in Table 2, where the first column gives the numbers of the curves in Figs. 4a and b. The values of the coefficients in formula (5) are determined to an accuracy of 10%.

Since the liquid heating curves for a Fourier number $F > 3.5 \cdot 10^{-3}$ are of a linear nature we can obtain from relationship (5) the vertical temperature distribution in the liquid for any instant, beginning at $F = 3.5 \cdot 10^{-3}$.

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